

Aproximace spojité funkce na intervalu – animace

Petr Vodstrčil

`petr.vodstrcil@vsb.cz`

Katedra aplikované matematiky, Fakulta elektrotechniky a informatiky,
Vysoká škola báňská–Technická univerzita Ostrava

24.3. 2015



Aproximace Lagrangeovými polynomy:

Nechť $a, b \in \mathbb{R}$, $a < b$, a $n \in \mathbb{N}$. Uvažujme ekvidistantní dělení

$$a = x_0 < x_1 < \cdots < x_n = b$$

intervalu $\langle a, b \rangle$.

Aproximace Lagrangeovými polynomy:

Nechť $a, b \in \mathbb{R}$, $a < b$, a $n \in \mathbb{N}$. Uvažujme ekvidistantní dělení

$$a = x_0 < x_1 < \cdots < x_n = b$$

intervalu $\langle a, b \rangle$.

Pro každé $n \in \mathbb{N}$ pak definujme operátor $L_n: C(\langle a, b \rangle) \rightarrow C(\langle a, b \rangle)$ předpisem

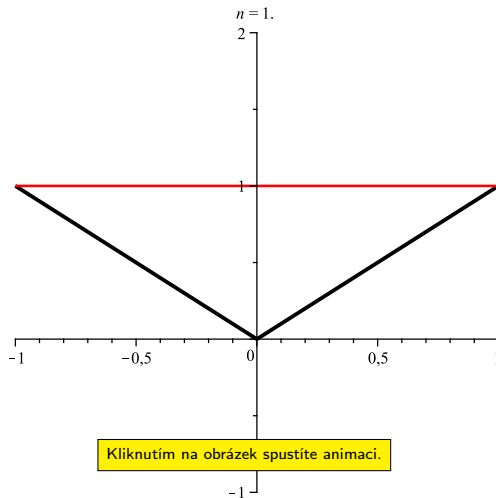
$$L_n(f)(x) = \underbrace{\sum_{k=0}^n f(x_k) \omega_k(x)}_{\text{Lagr. pol. } n\text{-tého řádu}},$$

kde ω_k ($k \in \{0, 1, \dots, n\}$) je polynom n -tého řádu splňující

$$\omega_k(x_i) = \begin{cases} 1 & \text{pro } i = k, \\ 0 & \text{pro } i \neq k. \end{cases}$$

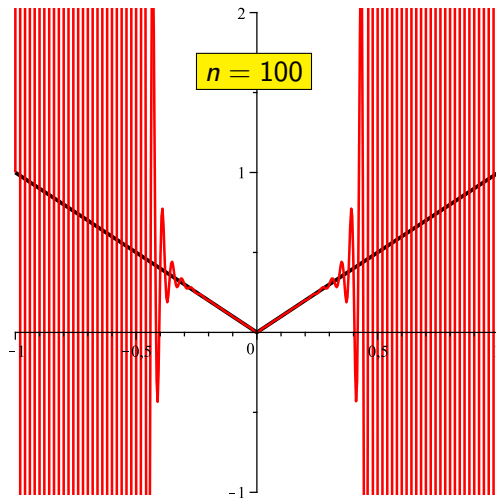
$$\left(L_n(f)(x) = \sum_{k=0}^n f(x_k) \omega_k(x) \right)$$

$$f(x) = |x|, \quad x \in \langle -1, 1 \rangle.$$



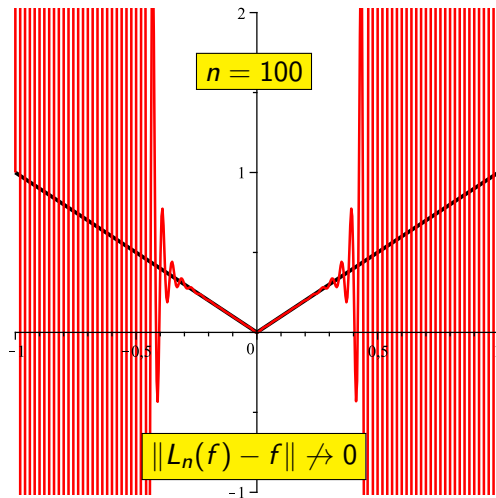
$$\left(L_n(f)(x) = \sum_{k=0}^n f(x_k) \omega_k(x) \right)$$

$$f(x) = |x|, \quad x \in \langle -1, 1 \rangle.$$



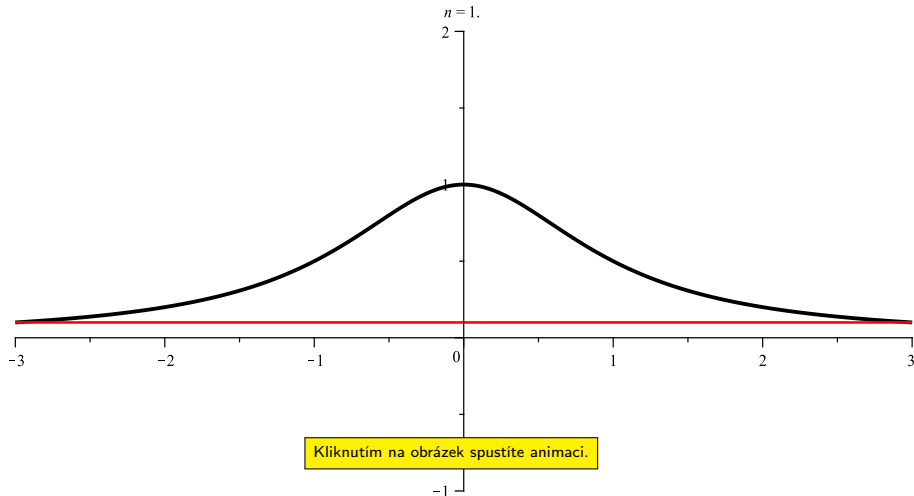
$$\left(L_n(f)(x) = \sum_{k=0}^n f(x_k) \omega_k(x) \right)$$

$$f(x) = |x|, \quad x \in \langle -1, 1 \rangle.$$



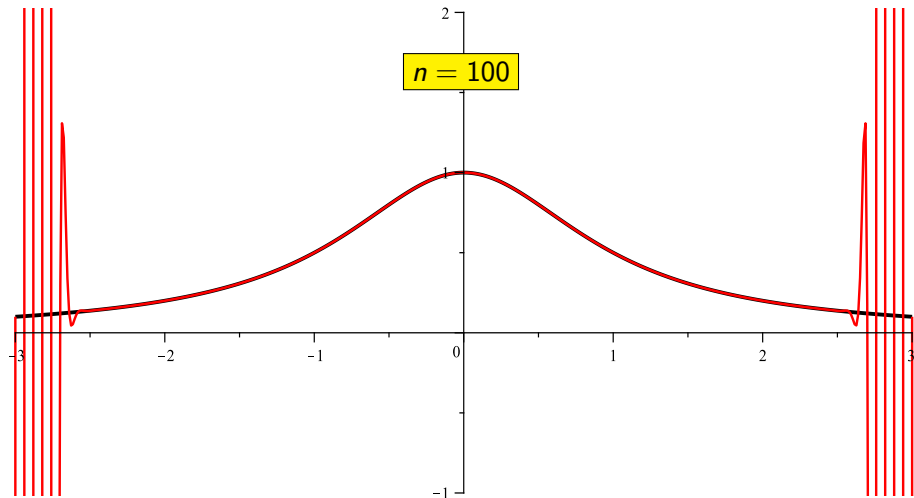
Aproximace Lagrangeovými polynomy: $\left(L_n(f)(x) = \sum_{k=0}^n f(x_k) \omega_k(x) \right)$

$$f(x) = \frac{1}{x^2 + 1}, \quad x \in \langle -3, 3 \rangle.$$



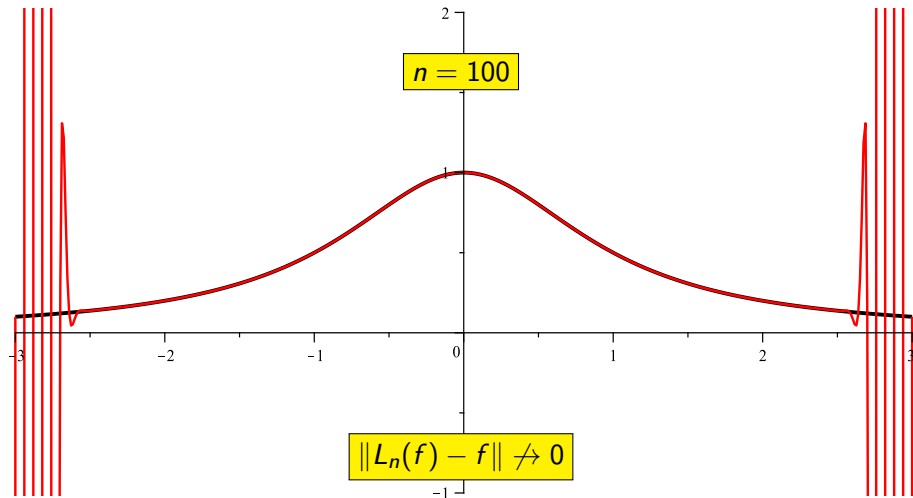
Aproximace Lagrangeovými polynomy: $\left(L_n(f)(x) = \sum_{k=0}^n f(x_k) \omega_k(x) \right)$

$$f(x) = \frac{1}{x^2 + 1}, \quad x \in \langle -3, 3 \rangle.$$



Aproximace Lagrangeovými polynomy: $\left(L_n(f)(x) = \sum_{k=0}^n f(x_k) \omega_k(x) \right)$

$$f(x) = \frac{1}{x^2 + 1}, \quad x \in \langle -3, 3 \rangle.$$



Aproximace Bernsteinovými polynomy:

Nechť $a, b \in \mathbb{R}$, $a < b$, a $n \in \mathbb{N}$. Uvažujme ekvidistantní dělení

$$a = x_0 < x_1 < \cdots < x_n = b$$

intervalu $\langle a, b \rangle$.

Aproximace Bernsteinovými polynomy:

Nechť $a, b \in \mathbb{R}$, $a < b$, a $n \in \mathbb{N}$. Uvažujme ekvidistantní dělení

$$a = x_0 < x_1 < \cdots < x_n = b$$

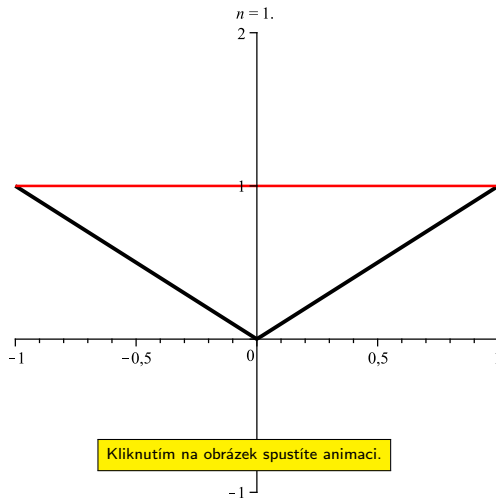
intervalu $\langle a, b \rangle$.

Pro každé $n \in \mathbb{N}$ pak definujme operátor $B_n: C(\langle a, b \rangle) \rightarrow C(\langle a, b \rangle)$ předpisem

$$B_n(f)(x) = \underbrace{\frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k}}_{\text{Bernsteinův polynom } n\text{-tého řádu}}.$$

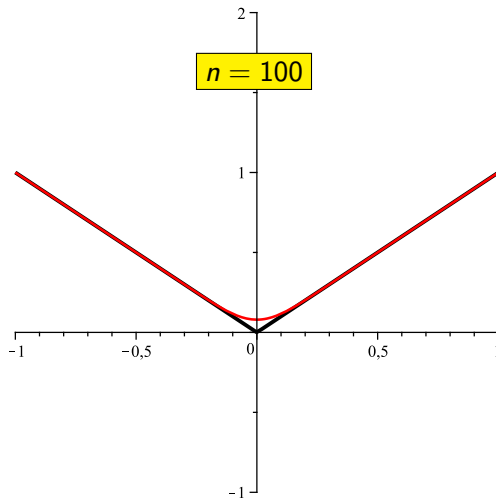
$$\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$$

$$f(x) = |x|, \quad x \in \langle -1, 1 \rangle.$$



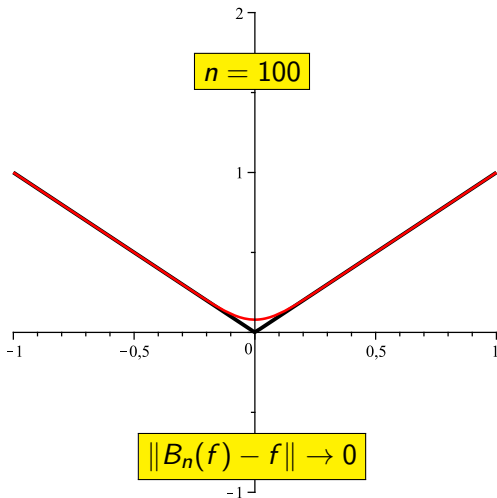
$$\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$$

$$f(x) = |x|, \quad x \in \langle -1, 1 \rangle.$$



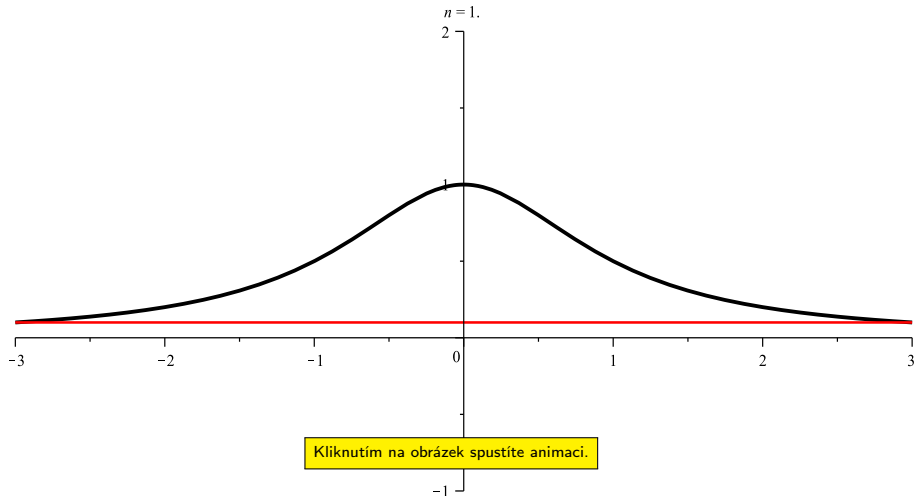
$$\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$$

$$f(x) = |x|, \quad x \in \langle -1, 1 \rangle.$$



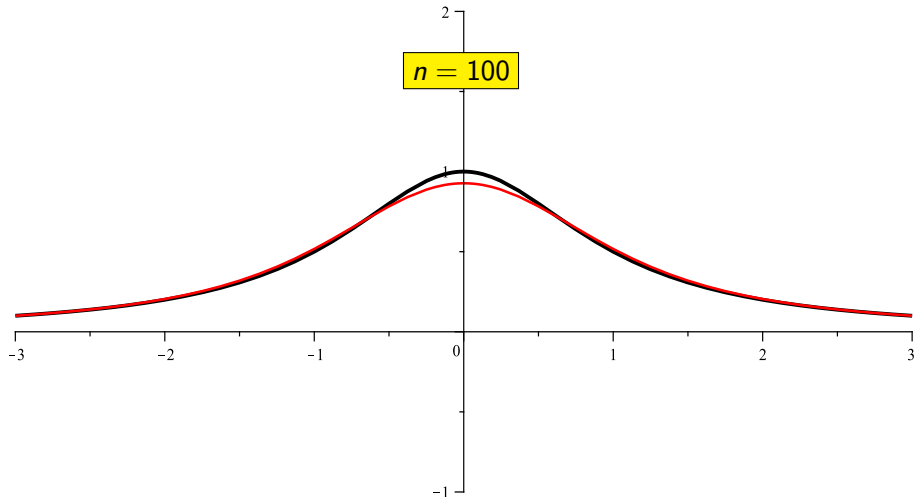
Aproximace Bernsteinovými polynomy: $\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$

$$f(x) = \frac{1}{x^2 + 1}, \quad x \in \langle -3, 3 \rangle.$$



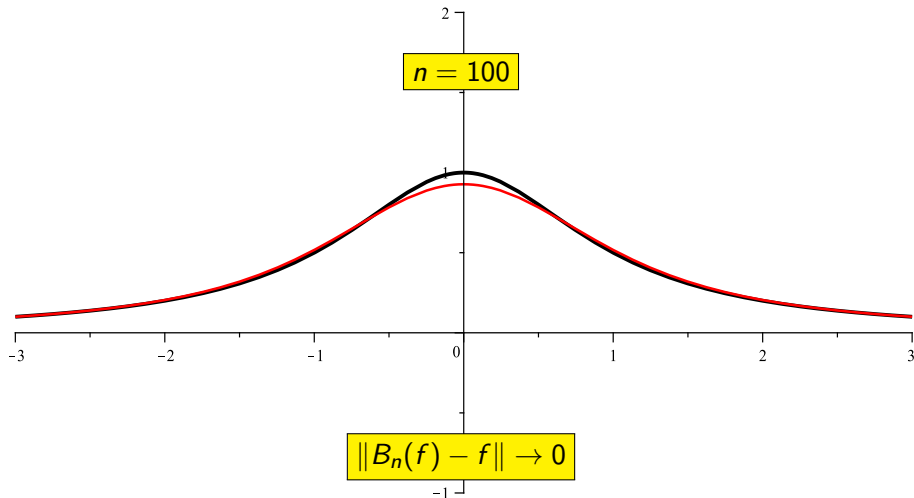
$$\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$$

$$f(x) = \frac{1}{x^2 + 1}, \quad x \in \langle -3, 3 \rangle.$$



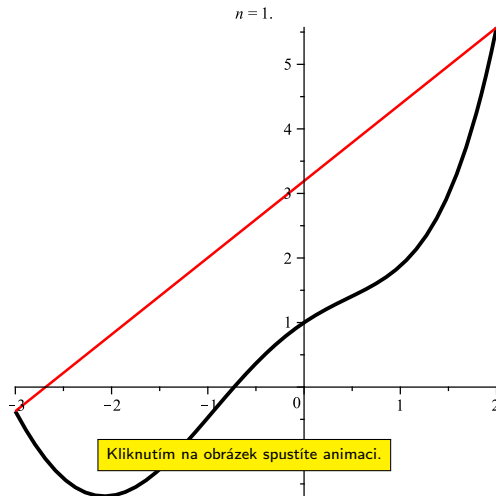
Aproximace Bernsteinovými polynomy: $\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$

$$f(x) = \frac{1}{x^2 + 1}, \quad x \in \langle -3, 3 \rangle.$$



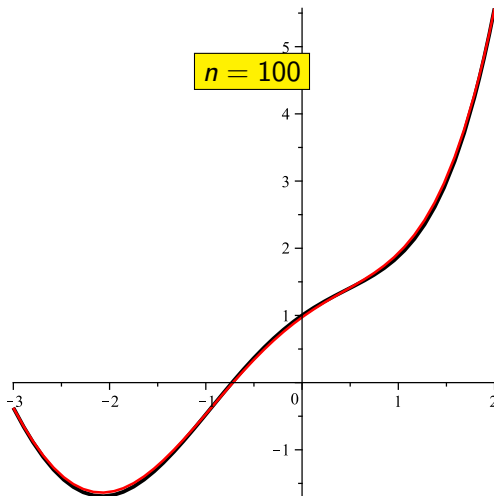
Aproximace Bernsteinovými polynomy: $\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$

$$f(x) = e^x - x \sin x, \quad x \in \langle -3, 2 \rangle.$$



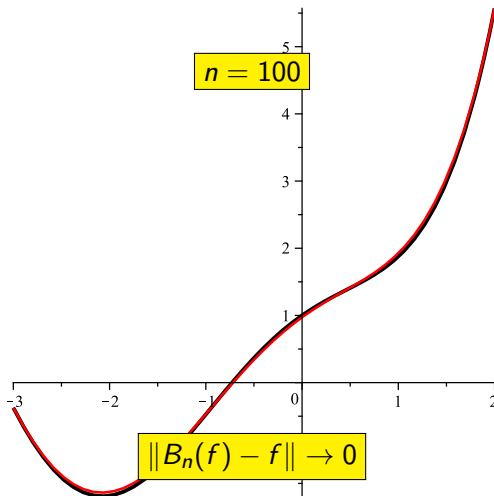
Aproximace Bernsteinovými polynomy: $\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$

$$f(x) = e^x - x \sin x, \quad x \in \langle -3, 2 \rangle.$$

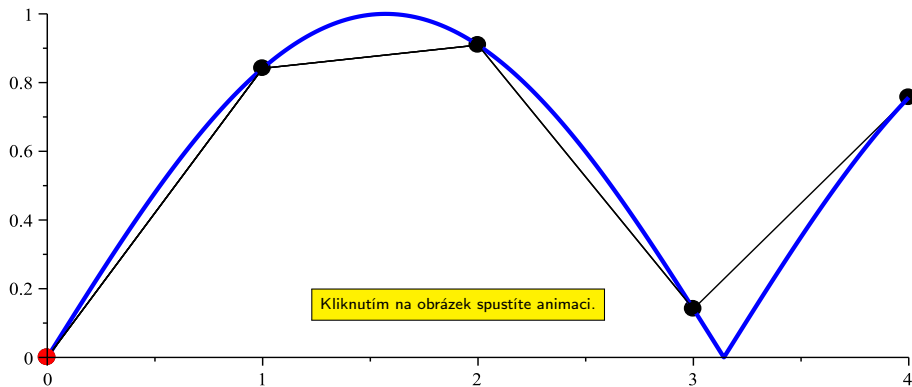


Aproximace Bernsteinovými polynomy: $\left(B_n(f)(x) = \frac{1}{(b-a)^n} \cdot \sum_{k=0}^n f(x_k) \binom{n}{k} (x-a)^k (b-x)^{n-k} \right)$

$$f(x) = e^x - x \sin x, \quad x \in \langle -3, 2 \rangle.$$



$$f(x) = |\sin x|, \quad x \in \langle 0, 4 \rangle.$$



$$f(x) = |\sin x|, \quad x \in \langle 0, 4 \rangle.$$

